HEAT AND MASS TRANSFER IN BUNDLES OF FINNED RODS

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A generalized relation for calculating the effective turbulent diffusion coefficients in bundles of finned rods is derived on the basis of an analysis of the results of interchannel cross-mixing studies by various authors.

In longitudinal flow past bundles of finned rods and twisted tubes having an oval profile, considerable intensification of the process of interchannel cross-mixing of the heat-transfer fluid (coolant) is observed in comparison with flow in a circular tube [1-13]. This fact is of utmost importance in equipment characterized by appreciable nonuniformity of the energy-release (heat-input) field in the cross section of the multirod bundle. The temperature distributions in bundles of finned rods are usually determined by the method of unit-cell computation with allowance for mass-, momentum-, and energy-transfer effects between the unit cells and with closure of the system of equations by the experimentally determined mixing coefficient μ [2-9]. In this case, however, enormous expenditures of computing time are required to implement the computer program for a large number of rods (tubes) in the bundle. Consequently, the temperature fields of the coolant in bundles of twisted tubes are determined by the method of homogenization of the real bundle [10-13]; this method is also recommended for calculating the temperature fields in bundles of finned rods.

A flow of a homogenized medium with a nonuniform energy-release field and with a coolant density that depends on the temperature and pressure can be described by the following system of equations [1] for the axisymmetrical problem:

$$\rho u \frac{\partial u}{\partial x} = -\frac{dP}{dx} + \frac{1}{r}\frac{\partial}{\partial r}\left(r\nu_{\text{ef}}\frac{\partial u}{\partial r}\right) - \xi\frac{\rho u^2}{2d_{\text{e}}}, \qquad (1)$$

$$G = 2\pi m \int_0^{r_{\text{K}}} \rho u r dr, \qquad (2)$$

$$\rho u c_p \frac{\partial T}{\partial x} = q_V \frac{1-m}{m} + \frac{1}{r}\frac{\partial}{\partial r}\left(r\lambda_{\text{ef}}\frac{\partial T}{\partial r}\right), \qquad (3)$$

$$P = \rho R T. \qquad (4)$$

The factor $(1 - m)/m$ in Eq. (3) takes the homogenization effect into account; m is the porosity of the bundle with respect to the coolant. The effective coefficients of turbulent viscosity ν_{ef} and thermal conductivity λ_{ef} in (1) and (3) can be expressed in terms of the effective turbulent diffusion coefficient D_t by assuming that the Lewis number Le_T and the Prandtl number Pr_T are equal to unity:

$$\lambda_{\text{ef}} = D_t \rho c_p, \qquad (5)$$

$$\nu_{\text{ef}} = \rho D_t. \qquad (6)$$

The coefficient D_t has been determined experimentally for bundles of twisted tubes [10-13]. This coefficient in the dimensionless form

$$k = \frac{D_t}{u d_{\text{e}}} \qquad (7)$$

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was related to the coefficient μ for bundles of finned rods having a porosity $m \approx 0.5$ [1]:

$$k = \mu p^2/4d_e \tag{8}$$

For the solution of the system of equations (1)-(4) it is augmented with the boundary conditions

$$u(0, r) = u_{in}, \quad T(0, r) = T_{in}, \quad P(0, r) = P_{in}, \tag{9}$$

$$\left.\frac{\partial u(x, r)}{\partial r}\right|_{r=r_K} = 0, \quad -\lambda \left.\frac{\partial T(x, r)}{\partial r}\right|_{r=r_K} = 0, \tag{10}$$

$$\left.\frac{\partial u(x, r)}{\partial r}\right|_{r=0} = 0, \quad \left.\frac{\partial T(x, r)}{\partial r}\right|_{r=0} = 0. \tag{11}$$

Methods and algorithms have now been developed for solving the system (1)-(4) with the boundary conditions (9)-(11) in order to calculate axisymmetrical temperature fields in multitube bundles [10-11]. The method of matrix factorization [12] can be used to calculate three-dimensional temperature fields as well.

Thus, the criterial dependence of the coefficient k on the governing similarity criteria (dimensionless groups) must be established in order for the homogenization method described here to be readily applicable to calculations of the temperature fields in bundles of finned rods with a porosity $m \neq 0.5$.

In published works [2-9] the porosity of bundles of finned rods with respect to the coolant varies in the limits $m \approx 0.27$-0.5. Therefore, instead of the relation (8) we use the following expression to find the dimensionless coefficient k from the experimentally determined coefficient μ:

$$k = \mu p^2 m/2d_e, \tag{12}$$

which is derived on the basis of the same considerations as the relation (8) [1].

Postulating that the nature of the flow in bundles of finned rods is similar to the flow in bundles of twisted tubes having an oval profile, we can assume that the transport properties of the flow in them are also practically identical. Then the criterial relation for the coefficient k can be sought in the form [13]:

$$k = k(\mathrm{Fr_M}, \mathrm{Re}, m, x/d_{ro}), \tag{13}$$

where

$$\mathrm{Fr_M} = S^2/d_{ro}d_e, \quad \mathrm{Re} = \rho u d_e/\mu_b.$$

In the present article we analyze and generalize eight studies of interchannel cross-mixing of the coolant in bundles of finned rods (or rods with helical wire wrapping) [2-9] using different methods for the experimental determination of the coefficient μ and different coolants. For example, the technique of heating the central rod ("thermal wake") is used in [2, 3, 6], an electromagnetic method is used in [4-6] (for liquid-metal heat-transfer media), and the authors of [7-9] describe a diffusion method based on the injection of warmer coolant into one of the cells with subsequent downstream measurement of the temperature distributions. Various coolants are used in these applications: air, water, liquid metals, and combinations thereof (see Table 1). In generalizing cross-mixing data it has been assumed that the different methods of investigation and coolant types used do not affect the numerical values of the coefficient μ or, accordingly, the coefficient k calculated according to (12). The geometrical parameters of the investigated bundles of finned rods, the experimental values of the coefficient μ, and the results of calculations of the governing criteria entering into (13) and the coefficient k are summarized in Table 1. In generalizing the cross-mixing data we have used the average values of the coefficient μ for the ranges of Reynolds numbers spanned by the experiments reported in [2-9]. This is explained by the fact that in the majority of cases the experiments were carried out for numbers $\mathrm{Re} > 10^4$ or the average values of Re at which μ was evaluated were greater than $\mathrm{Re} = 10^4$. This would explain the lack of any influence of Re on μ in the majority of papers surveyed. Experiments carried out in bundles of twisted tubes [13] have also shown that for $\mathrm{Re} \geq 10^4$ the coefficient k is practically

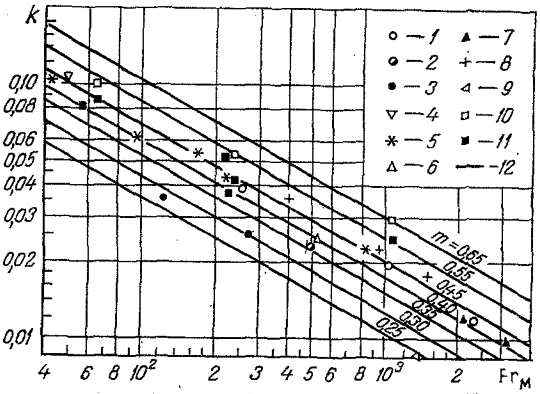

Fig. 1. Effective turbulent diffusion coefficient versus criterion Fr_M and porosity of the bundle with respect to the coolant. 1-4) Experimental data of [2-5] for various m; 5, 6) experimental data of [6] for m = 0.39, 0.326; 7) experimental data of [7, 8] for m = 0.41; 8, 9) experimental data of [9] for m = 0.39, 0.27; 10, 11) experimental data of [10, 12] and [11], respectively; 12) relation (16).

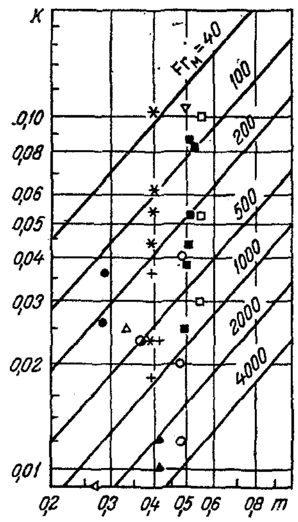

Fig. 2

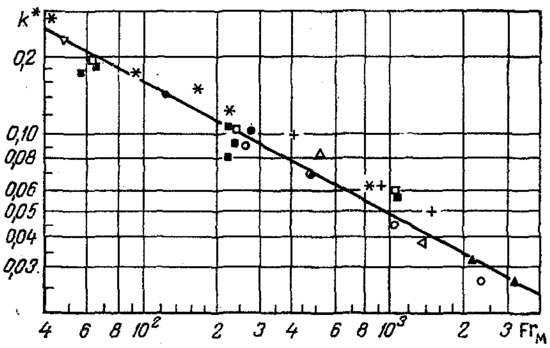

Fig. 3

Fig. 2. Effective turbulent diffusion coefficient versus porosity m for various numbers Fr_M (same nomenclature as in Fig. 1).

Fig. 3. Comparison of the criterial relation for the effective turbulent diffusion coefficient with the experimental data for bundles of finned rods and bundles of twisted tubes (same nomenclature as in Fig. 1).

independent of Re and the influence of Re on k is very slight for Re < 10^4. For Re < 10^4 the coefficient k in bundles of twisted tubes increases with the value of the number Re in accordance with the relation [13]

$$k = 3.1623\,[0.136\mathrm{Fr}_{\mathrm{M}}^{-0.256} + 10\mathrm{Fr}_{\mathrm{M}}^{-0.66}(m - 0.46)]\,\mathrm{Re}^{-0.125}. \quad (14)$$

To estimate the length of the initial section in bundles of finned rods in studying the cross-mixing process we used the formula derived in [13]:

$$x_{\mathrm{ib}}/d_{\mathrm{ro}} = 12.2a^{-1}\mathrm{Fr}_{\mathrm{M}}^{-0.275}, \quad (15)$$

where $a = 0.0745 + 11.37\mathrm{Fr}_{\mathrm{M}}^{-1} + 246\mathrm{Fr}_{\mathrm{M}}^{-2}$. The results of calculations according to (15) are given in Table 1, which also shows the relative lengths l/d_{ro} of the investigated bundles.

TABLE 1. Initial Geometrical Dimensions of Bundles of Finned Rods and Experimental Data on the Mixing Coefficient (bundles: Nos. 1-3 [2]; No. 4 [3]; Nos. 5-6 [4]; No. 7 [5]; Nos. 8-13 [6]; No. 14 [7]; No. 15 [8]; Nos. 16-19 [9])

Parameter	Bundle No.					
	1	2	3	4	5	6
Coolant	Na	Na	Na	Air	Na—K	Na—K
Spacer spacing S, mm	100	200	300	100	96	144
d_c, mm	6	6	6	6	19	19
d_{ro}, mm	9,84	9,84	9,84	8	21	21
S/d_{ro}	10,2	20,3	30,5	12,5	4,57	6,86
d_e, mm	3,96	3,96	3,96	2,58	3,63	3,63
S/d_e	25,3	50,5	75,8	38,8	26,7	40
$Fr_M=S^2/d_{ro}d_e$	258	1025	2310	485	122	274
$m=F_b/F_\Sigma$	0,476	0,476	0,476	0,36	0,28	0,28
Bundle length l, mm	1000	1000	1000	660	745	745
p, mm	7,92	7,92	7,92	7,02	20	20
μ, cm^{-1}	0,108	0,053	0,032	0,067	0,025	0,017
k	0,041	0,020	0,012	0,023	0,036	0,026
Re	$8\cdot10^3$—$7\cdot10^4$	$8\cdot10^3$—$7\cdot10^4$	$8\cdot10^3$—$7\cdot10^4$	$2\cdot10^4$—$1,5\cdot10^5$	$5\cdot10^3$—$2,3\cdot10^4$	$2,5\cdot10^3$—$2,1\cdot10^4$
Number of rods	61	61	61	61	19	19
x_{ib}/d_{ro}	21,6	21,08	18,1	22,4	17,6	21,8
l/d_{ro}	102	102	102	82,5	35,4	35,4

Parameter	Bundle No.					
	7	8	9	10	11	12
Coolant	Na—K	Na—K	Na—K	Na—K	Na—K	Na—K
Spacer spacing S, mm	144	96	144	192	192	375
d_c, mm	19	19	19	19	16	16,5
d_{ro}, mm	33,8	25,5	25,5	25,5	21,9	21,9
S/d_{ro}	4,26	3,76	5,65	7,53	8,77	17,12
d_e, mm	12,2	8,47	8,47	8,47	7,45	7,68
S/d_e	11,8	11,3	17	22,7	25,8	48,8
$Fr_M=S^2/d_{ro}d_e$	50	42,6	96	171	226	836
$m=F_b/F_\Sigma$	0,494	0,39	0,39	0,39	0,39	0,39
Bundle length l, mm	745	1000	1000	1000	1824	701
p, mm	26,4	22,23	22,23	22,23	18,96	19,55
μ, cm^{-1}	0,073	0,092	0,055	0,048	0,047	0,024
k	0,103	0,105	0,063	0,055	0,044	0,023
Re	$4\cdot10^3$—$3,4\cdot10^4$	$2,8\cdot10^4$—$4\cdot10^4$	$4\cdot10^3$—$3,5\cdot10^4$	$2,8\cdot10^4$—$4\cdot10^4$	$6\cdot10^3$—$5\cdot10^4$	$2\cdot10^3$—$5\cdot10^4$
Number of rods	19	19	19	19	37	37
x_{ib}/d_{ro}	10,4	9,1	15,8	20	21,1	21,5
l/d_{ro}	22	39	39	39	83	32

Parameter	Bundle No.						
	13	14	15	16	17	18	19
Coolant	Na—K	H_2O	H_2O	H_2O	H_2O	H_2O	H_2O
Spacer spacing S, mm	100	300	255	300	450	575	450
d_c, mm	6,1	5,84	6,3	21	21	21	23
d_{ro}, mm	7,93	8,76	9,07	26,9	26,9	26,9	25,8
S/d_e	12,6	34,3	28,1	11,2	16,74	21,4	17,5
d_e, mm	2,41	3,2	3,35	8,3	8,3	8,3	5,64
S/d_e	41,5	93,8	76,1	36,3	54,4	69,5	79,8
$Fr_M=S^2/d_{ro}d_e$	523	3210	2140	405	911	1487	1394
$m=F_b/F_\Sigma$	0,326	0,41	0,41	0,39	0,39	0,39	0,27
Bundle length l, mm	1000	1450	1796	800	800	800	800
p, mm	7,02	7,3	7,68	23,94	23,94	23,94	24,4
μ, cm^{-1}	0,076	0,03	0,034	0,027	0,017	0,0133	0,0063
k	0,025	0,0102	0,0122	0,036	0,023	0,018	0,009
Re	$2\cdot10^3$—$1,2\cdot10^4$	$1\cdot10^4$—$3\cdot10^4$	$4\cdot10^3$—$1,5\cdot10^4$	$2,5\cdot10^4$—$5,2\cdot10^4$	$4,2\cdot10^4$	$4,2\cdot10^4$	$4,2\cdot10^4$
Number of rods	37	217	91	7	7	7	7
x_{ib}/d_{ro}	22,3	16,9	18,5	22,3	21,3	19,7	20,1
l/d_{ro}	126	165	198	29,8	29,8	29,8	31

It is seen that $l/d_{ro} > x_{ib}/d_{ro}$ for all the bundles and is quite large in the majority of cases. We can therefore assume that the coefficient k depends for the most part only on the number Fr_M and the porosity m of the bundle with respect to the coolant:

$$k = k(Fr_M,\ m).$$

Then the experimental data given in Table 1 can be generalized by the criterial relation

$$k = 1.902 Fr_M^{-0.53} m^{1.086}. \qquad (16)$$

TABLE 2. Experimental Data on the Mixing Coefficient in Bundles of Twisted Tubes (bundles: Nos. 1-3 [10, 12]; Nos. 4-8 [11]; coolant: air)

Parameter	Bundle No.								
	1	2	3	4	5	6	7	8	9
Bundle porosity	0,544	0,539	0,527	0,477	0,492	0,496	0,51	0,51	0,49
Fr_M	63,5	232	1050	1080	236	65	57	222	222
k	0,10	0,053	0,03	0,025	0,043	0,087	0,083	0,053	0,037

The graph in Fig. 1, plotted in coordinates $k = f(Fr_M)$, exhibits stratification of the experimental data according to the porosity m, and the graph in Fig. 2 in coordinates $k = \varphi(m)$ shows the same according to the number Fr_M, i.e., the coefficient k increases with increasing porosity m and decreases with increasing value of Fr_M. If the experimental data are compared with (16) in the form

$$k^* = km^{-1.086} = f(Fr_M),$$

we infer from Fig. 3 that all the experimental points are well grouped about the line representing expression (16).

Thus, Eq. (16) can be used in the ranges $Fr_M = 43\text{-}3300$ and $m \geq 0.27$ to calculate the coefficient k needed in order to close the system of equations (1)-(4) with application of the homogenized flow model to determine the temperature fields of the coolant in bundles of finned rods for a nonuniform energy-release field in the bundle cross section.

Also in good agreement with the relation (16) are experimental data obtained on the coefficients k for bundles of twisted tubes having an oval profile for $Re \geq 10^4$ [10-13] (Figs. 1-3). The governing similarity criteria and values of the coefficient k in [10-13] are shown in Table 2. Consequently, in the zone of self-similarity of the coefficient k with respect to the Reynolds number ($Re \geq 10^4$) the single criterial relation (16) can be used to describe the process of interchannel cross-mixing in bundles of finned rods and also in bundles of twisted tubes having an oval profile. This fact evinces the identical mechanism underlying the heat- and mass-transfer processes in such bundles.

NOTATION

$\mu = G_{ij}/G_i$, mixing coefficient; G_{ij}, mass flow of coolant in transverse direction from i-th to j-th cell per unit length of channel; G_i, axial mass flow of coolant in i-th cell; T, temperature; u, velocity; ρ, density; P, pressure; x, r, coordinate; ξ, hydraulic friction coefficient; c_p, specific heat; q_V, volumetric heat release; λ, thermal conductivity; Fr_M, similarity criterion characterizing the action of centrifugal forces on the flow in the multirod bundle; d_c, diameter of cylindrical part of rod; d_{ro}, diameter of rod plus spacers; d_e, equivalent diameter of multirod bundle; F_b, transfer cross section of bundle; F_Σ, cross-sectional area of channel with rods; p, spacing of rod array.

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SOLUTION OF A PROBLEM ON TWO-SIDED HEAT EXCHANGE IN HEAT EXCHANGERS WITH A TWISTED FLOW

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Analytical relations are obtained for the temperature profile of heat carriers along the flow axis in a heat exchanger with an inner spiralled tube in the case of two-sided heating.

A problem that is presently very important is reducing the weight and dimensions of a heat exchanger widely used in different areas of technology. An effective way of solving this problem is developing compact heat-exchange surfaces.

An example of an apparatus with a high energy intensity is a heat exchanger with two-sided heating and elements in the form of "tube-in-tube" channels with a spiral inner tube, where part of the outside of the spiral tube is in contact with the outer tube along the helix. The heating medium thus moves inside the inner tube and between the tubes, while the heated medium moves in the spiral channel formed between the surfaces of the spiralled inner tube and the outer tube.

Such a design increases the compactness coefficient by a factor of 1.5-1.8 compared to straight-tube heat exchangers with one-sided heating, while the spiralling of the inner tube helps intensify heat transfer both on the side of the heating medium and on the side of the heated medium as a result of twisting of the flow of heat carriers.

Existing methods of calculating the temperature fields of heat exchangers with two-sided heating [1-3] do not take into account heat exchange which occurs between the flows of the heating heat carrier at the site of contact of the spiralled inner tube and the outer tube over the entire length of the element.

This article presents analytical relations which make it possible, given prescribed boundary conditions, to obtain the distribution not only of the outlet temperatures, but also

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